MECHANICAL VIBRATIONS

Course Name: B.Tech-ME

Semester: 7th

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UNIT: I

Introduction

Mechanical vibration ⇨

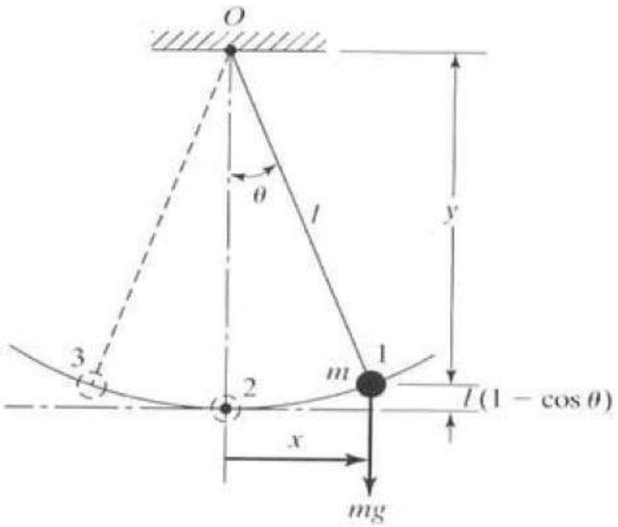
- Defined as oscillatory motion of bodies in response to disturbance.
- Oscillations occur due to the presence of a restoring force
- Vibrations are everywhere:
  - Human body: eardrums, vocal cords, walking and running
  - Vehicles: residual imbalance of engines, locomotive wheels
  - Rotating machinery: Turbines, pumps, fans, reciprocating machines
  - Musical instruments
- Excessive vibrations can have detrimental effects:
  - Noise
  - Loosening of fasteners
  - Tool chatter
  - Fatigue failure
  - Discomfort
- When vibration frequency coincides with natural frequency, resonance occurs.

Fundamentals

- In simple terms, a vibratory system involves the transfer of potential energy to kinetic energy and vice-versa in alternating fashion.
- When there is a mechanism for dissipating energy (damping) the oscillation gradually diminishes.
- In general, a vibratory system consists of three basic components:
  - A means of storing potential energy (spring, gravity)
  - A means of storing kinetic energy (mass, inertial component)
  - A means to dissipate vibrational energy (damper)
  - This can be observed with a pendulum:
  - At position 1: the kinetic energy is zero and the potential energy is

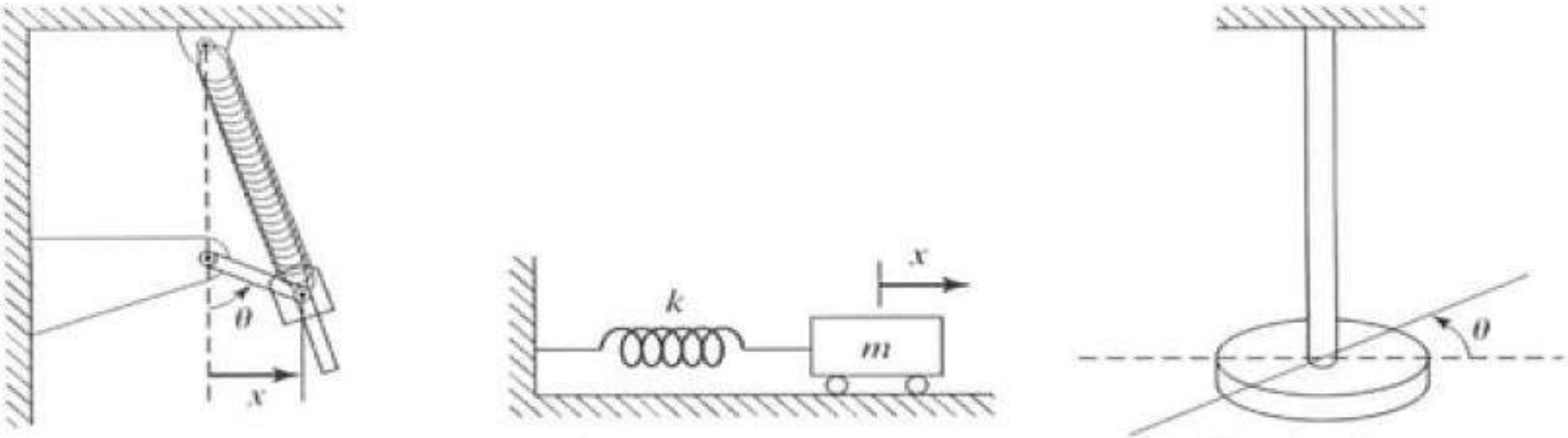
$$mgl(1-\cos\theta)$$

  - At position 2: the kinetic energy is at its maximum
  - At position 3: the kinetic energy is again zero and the potential energy at its maximum.



Degree of Freedom

- The number of degrees of freedom : number of independent coordinates required to completely determine the motion of all parts of the system at any time.
- Examples of single degree of freedom systems:



(a) Slider-crank-spring mechanism (b) Spring-mass system (c) Torsional system

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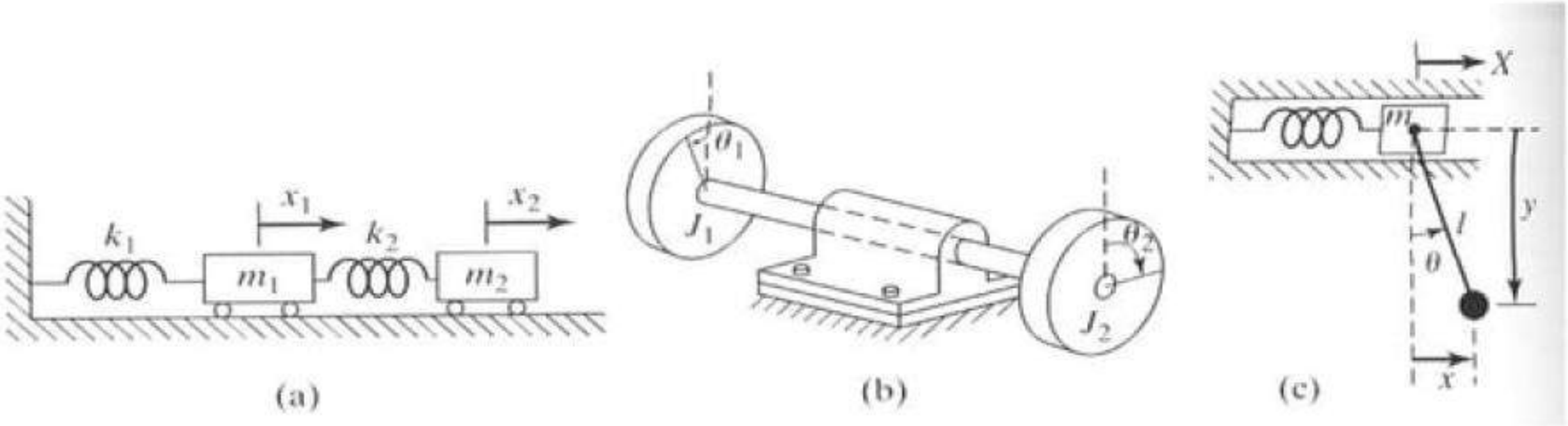
Degree of Freedom

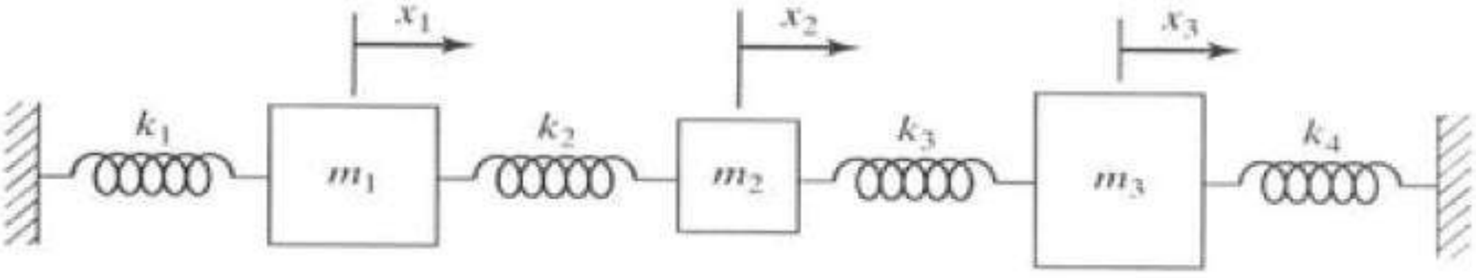
- Examples of two degree of freedom systems:



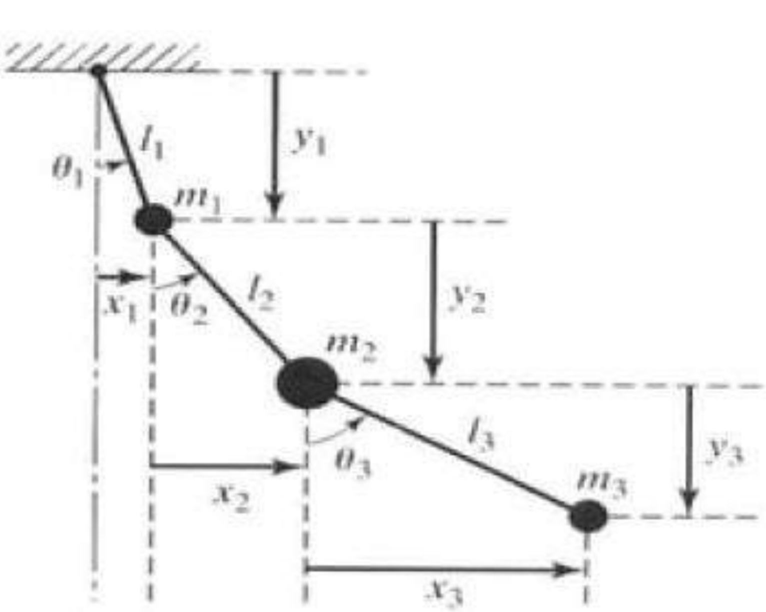
(a) (b) (c)

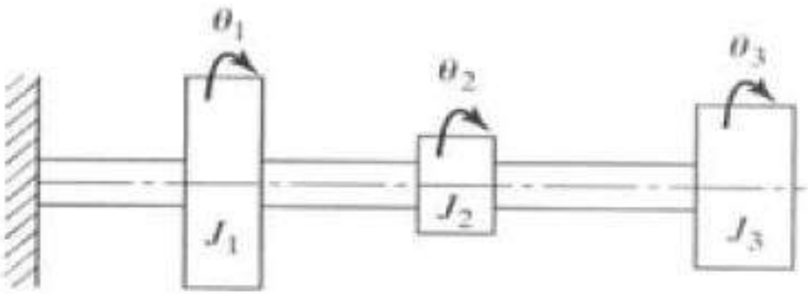
Degree of Freedom

- Examples of three degree of freedom systems:



(a)

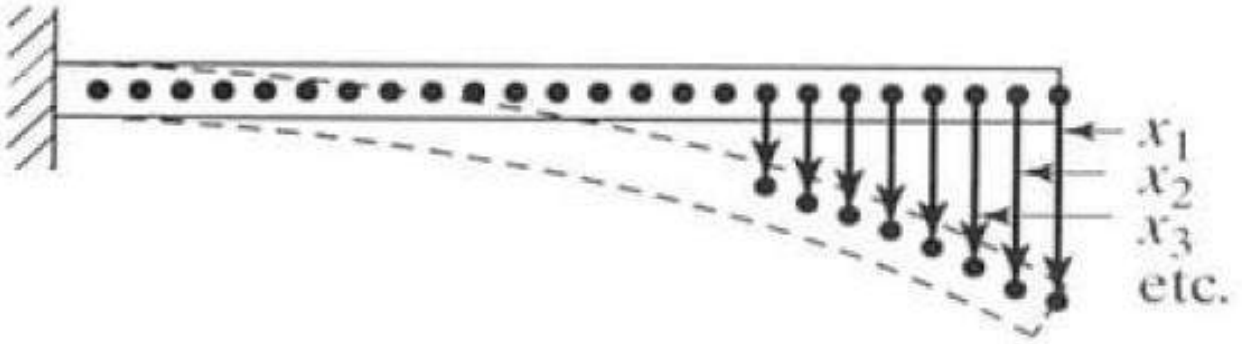




(c)

Discrete and Continuous System

- Many practical systems small and large or structures can be describe with a finite number of DoF. These are referred to as *discrete* or *lumped* parameter systems
- Some large structures (especially with continuous elastic elements) have an infinite number of DoF These are referred to as *continuous* or *distributed* systems.
- In most cases, for practical reasons, continuous systems are approximated as discrete systems with sufficiently large numbers lumped masses, springs and dampers. This equates to a large number of degrees of freedom which affords better accuracy.



Classification of Vibrations

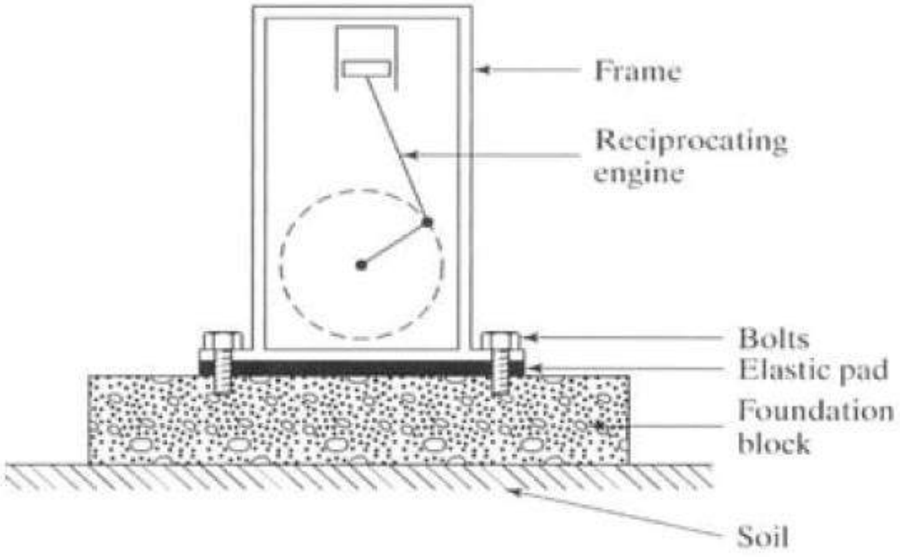
- Free and Forced vibrations
  - *Free vibration*: Initial disturbance, system left to vibrate without influence of external forces.
  - *Forced vibration*: Vibrating system is stimulated by external forces. If *excitation* frequency coincides with *natural* frequency, resonance occurs.

- Undamped and damped vibration
  - *Undamped vibration*: No dissipation of energy. In many cases, damping is (negligibly) small (steel 1 – 1.5%). However small, damping has critical importance when analysing systems at or near resonance.
  - *Damped vibration*: Dissipation of energy occurs - vibration amplitude decays.

- Linear and nonlinear vibration
  - *Linear vibration*: Elements (mass, spring, damper) behave linearly. Superposition holds - double excitation level = double response level, mathematical solutions well defined.
  - *Nonlinear vibration*: One or more element behave in nonlinear fashion (examples). Superposition does not hold, and analysis technique not clearly defined.

Vibration Analysis

- Input (excitation) and output (response) are wrt time
- Response depend on ***initial*** conditions and external forces
- Most practical systems very complex – (mathematical) modelling requires simplification
- Procedure:
  - → Mathematical modelling
  - → Derivation / statement of governing equations
  - → Solving of equations for specific boundary conditions and external forces
  - → Interpretation of solution(s)



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Spring Element

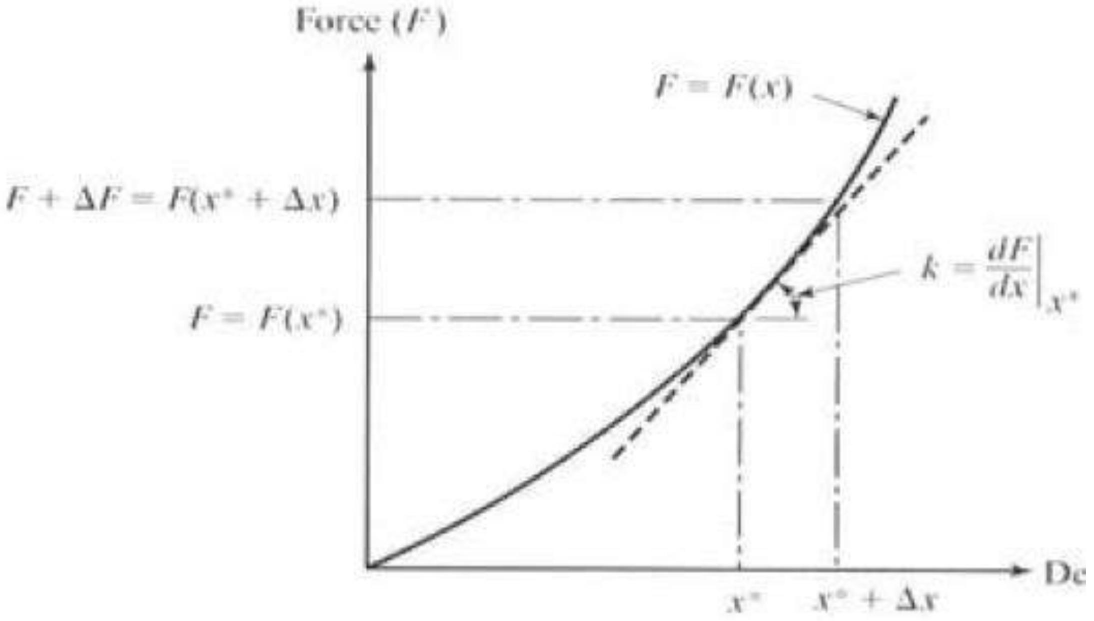
- Pure spring element considered to have negligible mass and damping
- Force proportional to spring deflection (relative motion between ends):

$$F = k\Delta x$$

- For linear springs, the potential energy stored is:

$$U = \frac{1}{2}k(\Delta x)^2$$

- Actual springs sometimes behave in nonlinear fashion
- Important to recognize the presence and significance (magnitude) of nonlinearity
- Desirable to generate linear estimate



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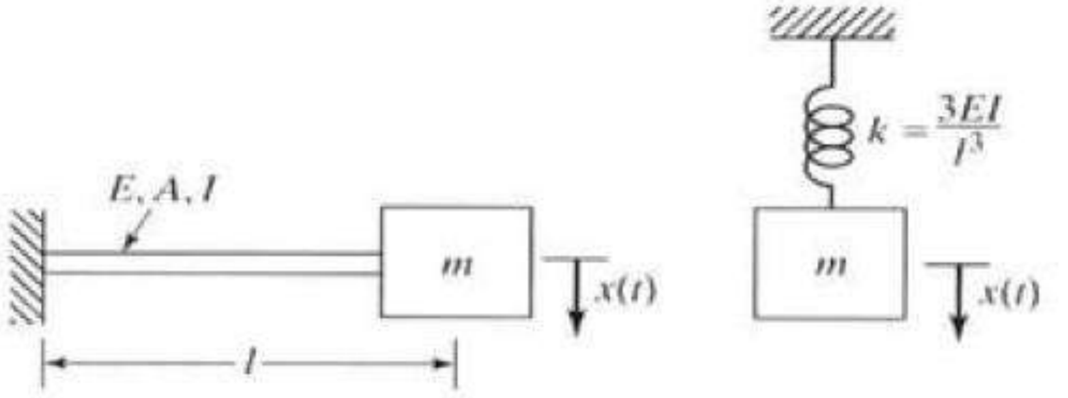
Spring Element

- Equivalent spring constant.
  - Eg: cantilever beam: Mass of beam assumed negligible cf lumped mass
  - Deflection at free end:

$$\delta = \frac{mgl^3}{3EI}$$

  - Stiffness (Force/defln):

$$k = \frac{mg}{\delta} = \frac{3EI}{l^3}$$



(a) Actual system
(b) Single degree of freedom model

- This procedure can be applied for various geometries and boundary conditions. (see appendix)

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Spring Element

- Equivalent spring constant.
  - Springs in parallel:

$$w = mg = k_1\delta + k_2\delta$$

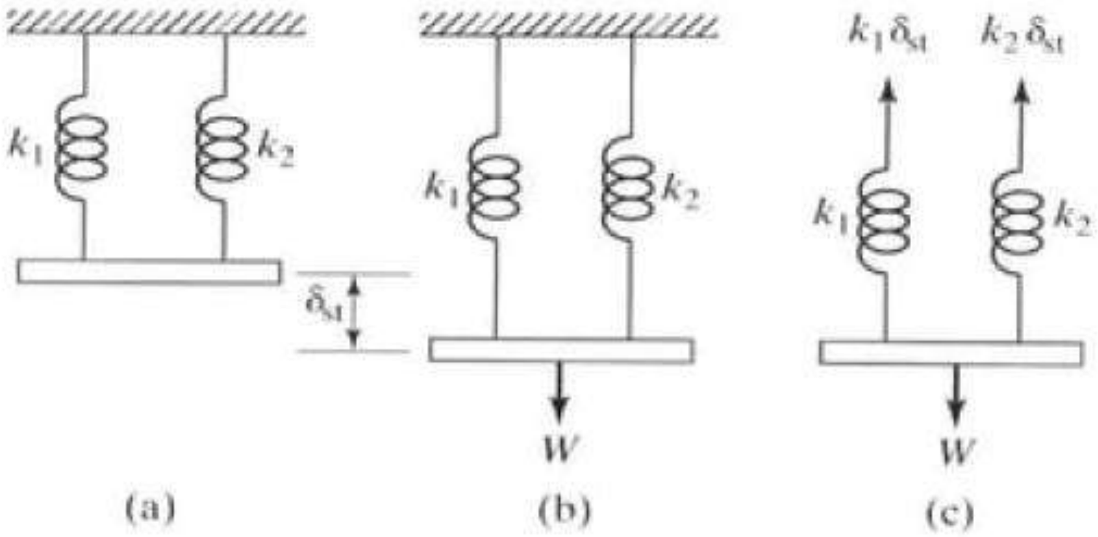
$$w = mg = k_{eq}\delta$$

- where

$$k_{eq} = k_1 + k_2$$

- In general, for n springs in parallel:

$$k_{eq} = \sum_{i=1}^{i=n} k_i$$



Spring Element

- Equivalent spring constant.
  - **Springs in series:**

$$\delta_t = \delta_1 + \delta_2$$

- Both springs are subjected to the same force:

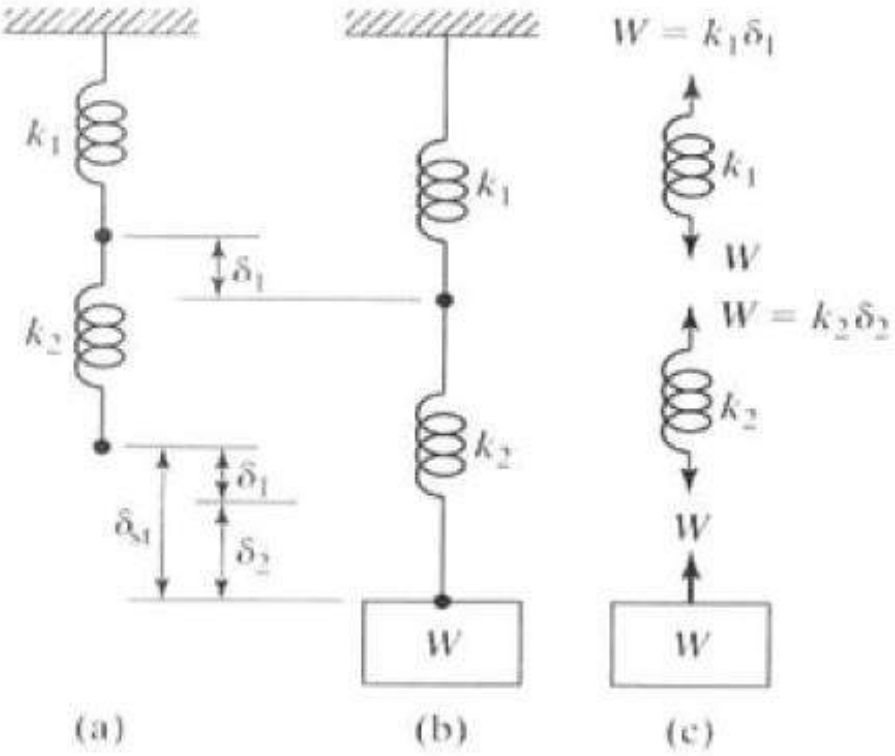
$$mg = k_1\delta_1 = k_2\delta_2$$

$$mg = k_{eq}\delta_t$$

- Combining the above equations:

$$k_1\delta_1 = k_2\delta_2 = k_{eq}\delta_t$$

$$\delta_1 = \frac{k_{eq}\delta_t}{k_1} \quad and \quad \delta_2 = \frac{k_{eq}\delta_t}{k_2}$$



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Spring Element

- **Springs in series (cont'd):**
  - Substituting into first eqn:

$$\delta_t = \frac{k_{eq}\delta_t}{k_1} + \frac{k_{eq}\delta_t}{k_2}$$

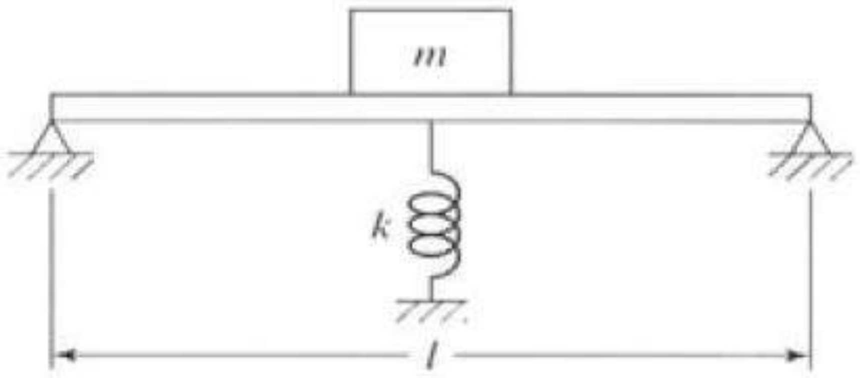
  - Dividing by $k_{eq}\delta_t$ throughout:

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

- For n springs in series:

$$\frac{1}{k_{eq}} = \sum_{i=1}^{i=n}\left[\frac{1}{k_i}\right]$$

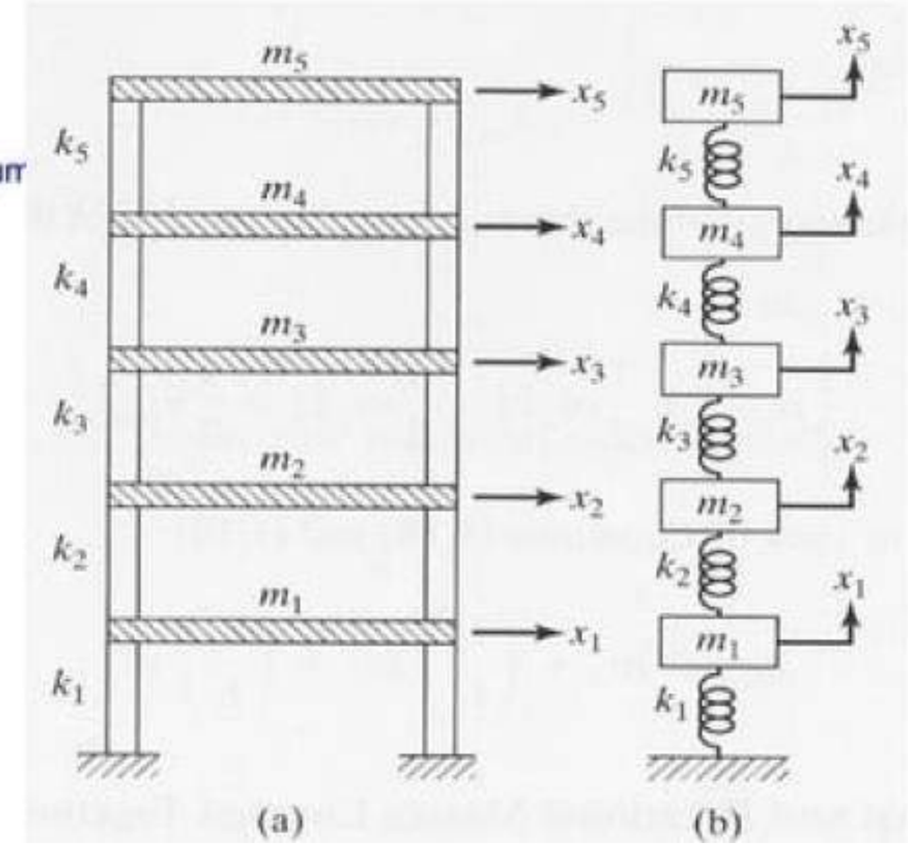
  - Equivalent spring constant.
    - When springs are connected to rigid components such as pulleys and gears, the energy equivalence principle must be used.
  - Example:



Mass/Inertia Element

- Mass or inertia element assumed rigid (lumped mass)
- Its energy (kinetic) is proportional to velocity.
- Force ∝ mass * acceleration
- Work = force * displacement
- Work done on mass is stored as Kinetic Energy

- Modelling with lumped mass elements. Example: assum frame mass is negligible cf mass of floors.



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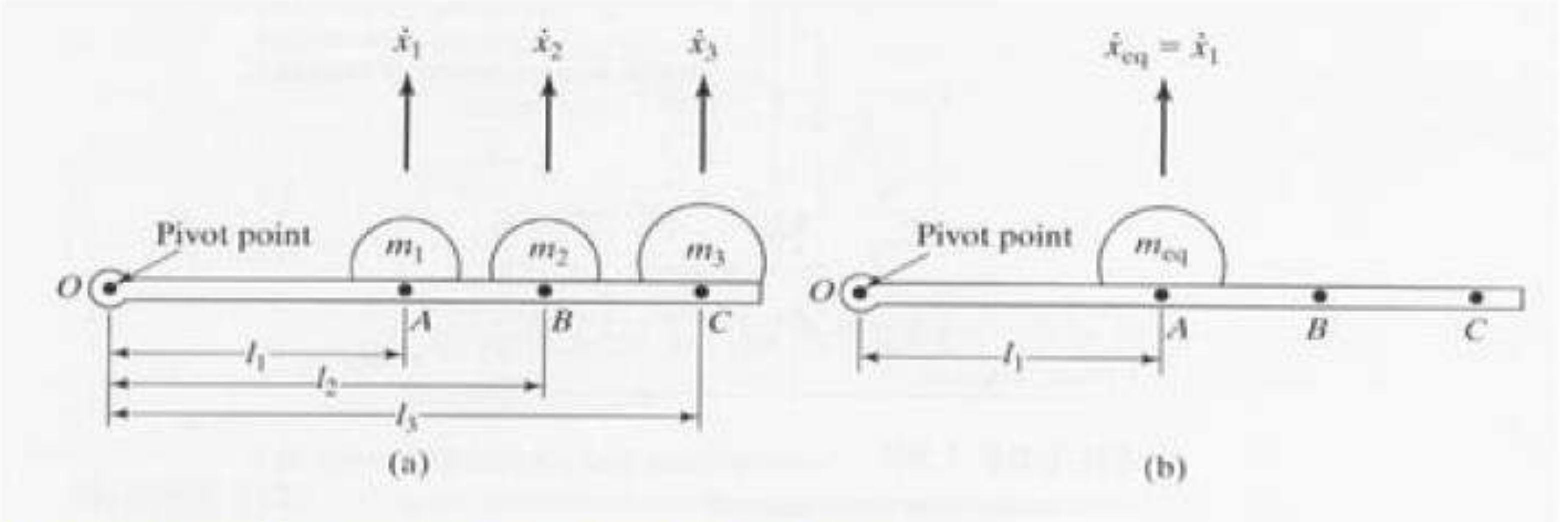
Mass/Inertia Element

- Equivalent mass - example:



- The velocities of the mass elements can be written as:

$$\dot{x}_2 = \frac{l_2}{l_1}\dot{x}_1 \quad and \quad \dot{x}_3 = \frac{l_3}{l_1}\dot{x}_1$$

- To determine the equivalent mass at position l_1:

$$\dot{x}_{eq} = \dot{x}_1$$

- Equivalent mass – example (cont'd)

- Equating the kinetic energies:

$$\frac{1}{2}m_1\dot{x}_1^2 + \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}m_3\dot{x}_3^2 = \frac{1}{2}m_{eq}\dot{x}_{eq}^2$$

- Substituting for the velocity terms:

$$m_{eq} = m_1 + \left(\frac{l_2}{l_1}\right)^2 m_2 + \left(\frac{l_3}{l_1}\right)^2 m_3$$

Damping Element

- Absorbs energy from vibratory system → vibration amplitude decays.
- Damping element considered to have no mass or elasticity
- Real damping systems very complex, damping modelled as:

  - **Viscous damping:**
    - Based on viscous fluid flowing through gap or orifice.
    - Eg: film between sliding surfaces, flow b/w piston & cylinder, flow thru orifice, film around journal bearing.
    - Damping force ∝ relative velocity between ends

  - **Coulomb (dry Friction) damping:**
    - Based on friction between unlubricated surfaces
    - Damping force is constant and opposite the direction of motion

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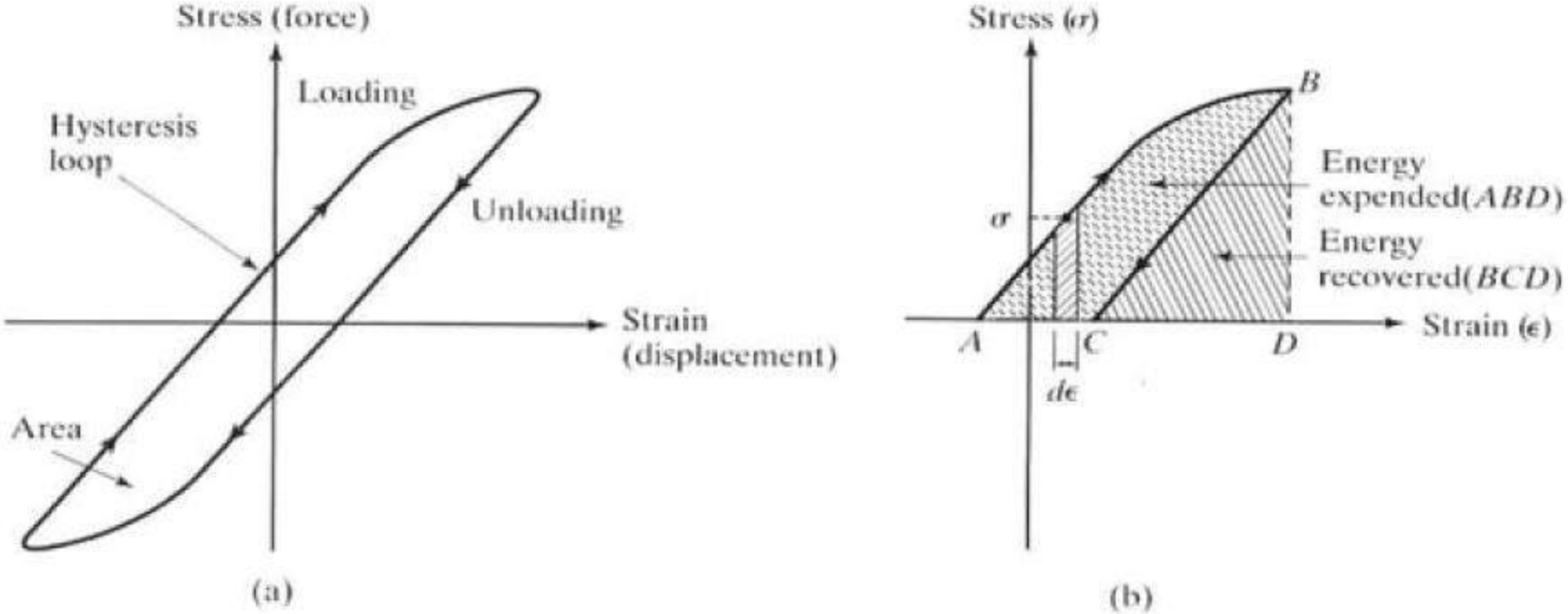
Damping Element

- **Hysteretic (material or solid) damping:**
  - Based on plastic deformation of materials (energy loss due to slippage b/w grains)
  - Energy lost due to hysteresis loop in force-deflection (stress-strain) curve of element when load is applied:



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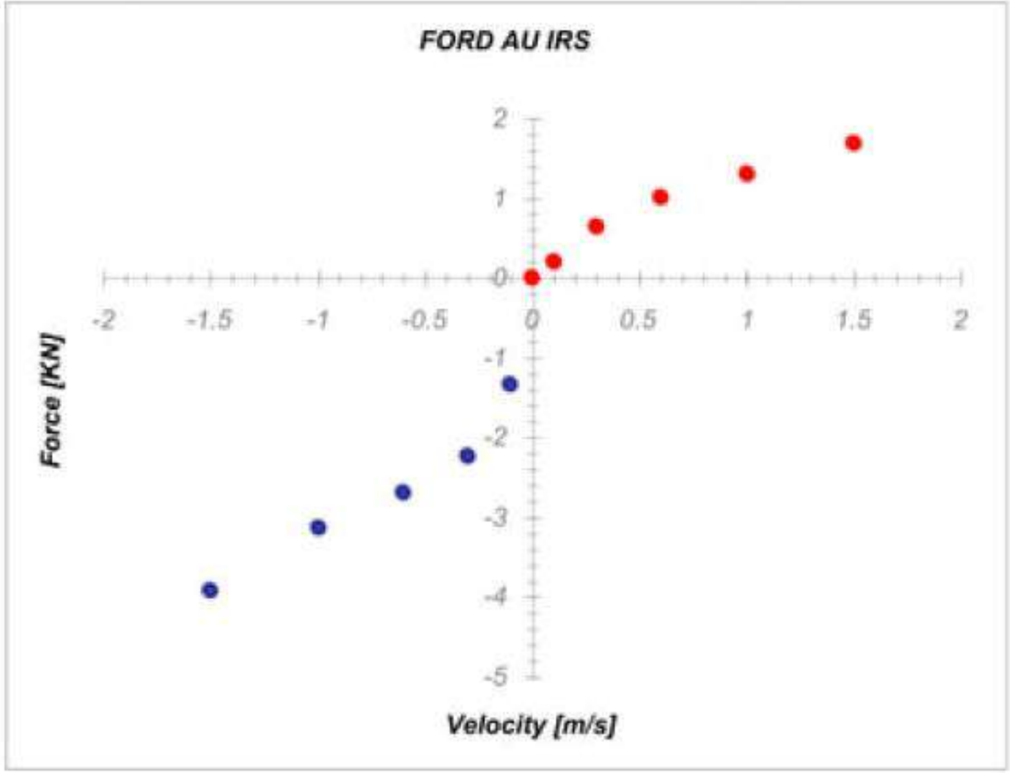
Damping Element

- **Equivalent damping element:**
  - Combinations of damping elements can be replace by equivalent damper using same procedures as for spring and mass/inertia elements.



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Harmonic Motion

- Harmonic motion: simplest form of periodic motion (deterministic).
- Pure sinusoidal (co-sinusoidal) motion
- Eg: Scotch-yoke mechanism rotating with angular velocity ω - simple harmonic motion:
- The motion of mass m is described by:

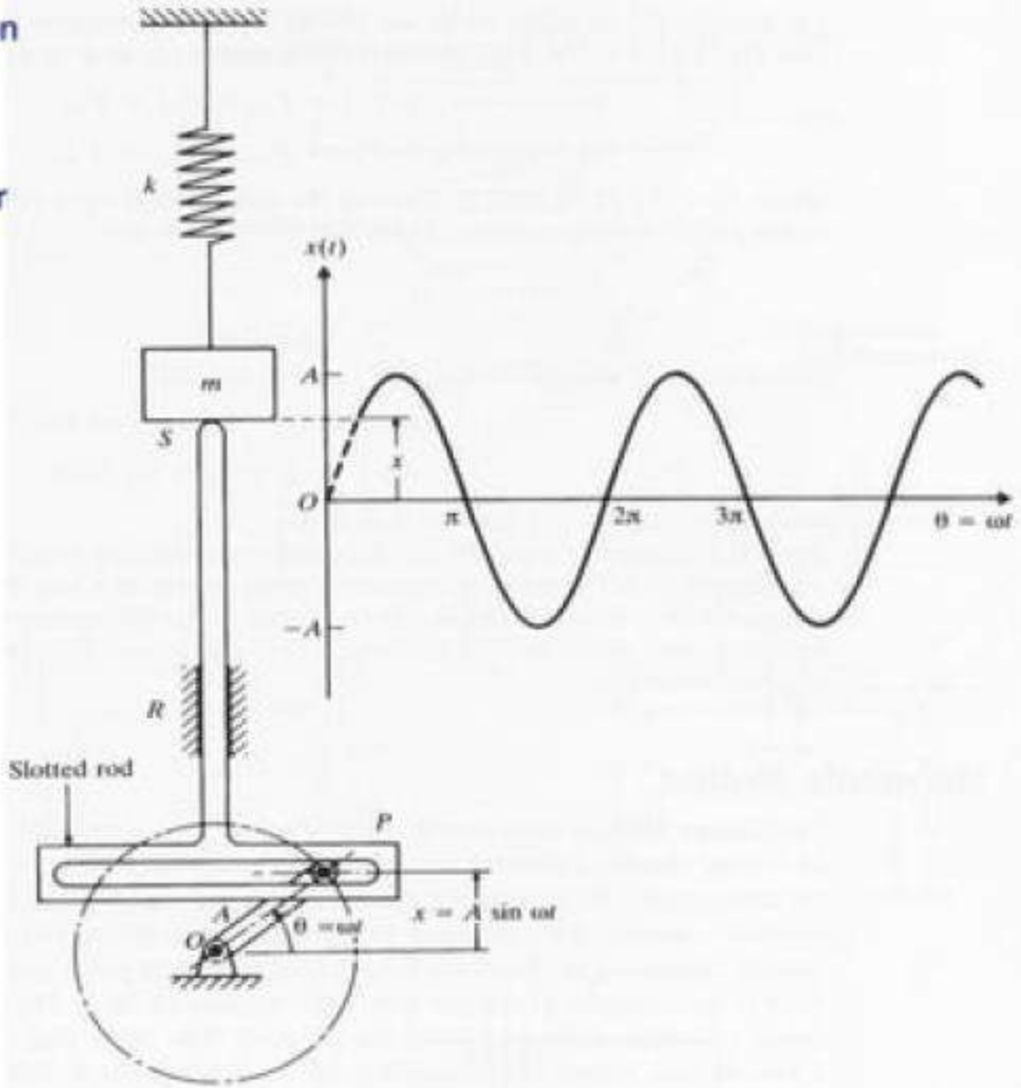
$$x = A\sin(\theta) = A\sin(\omega t)$$

- Its velocity and acceleration are:

$$\frac{dx}{dt} = \omega A\cos(\omega t)$$

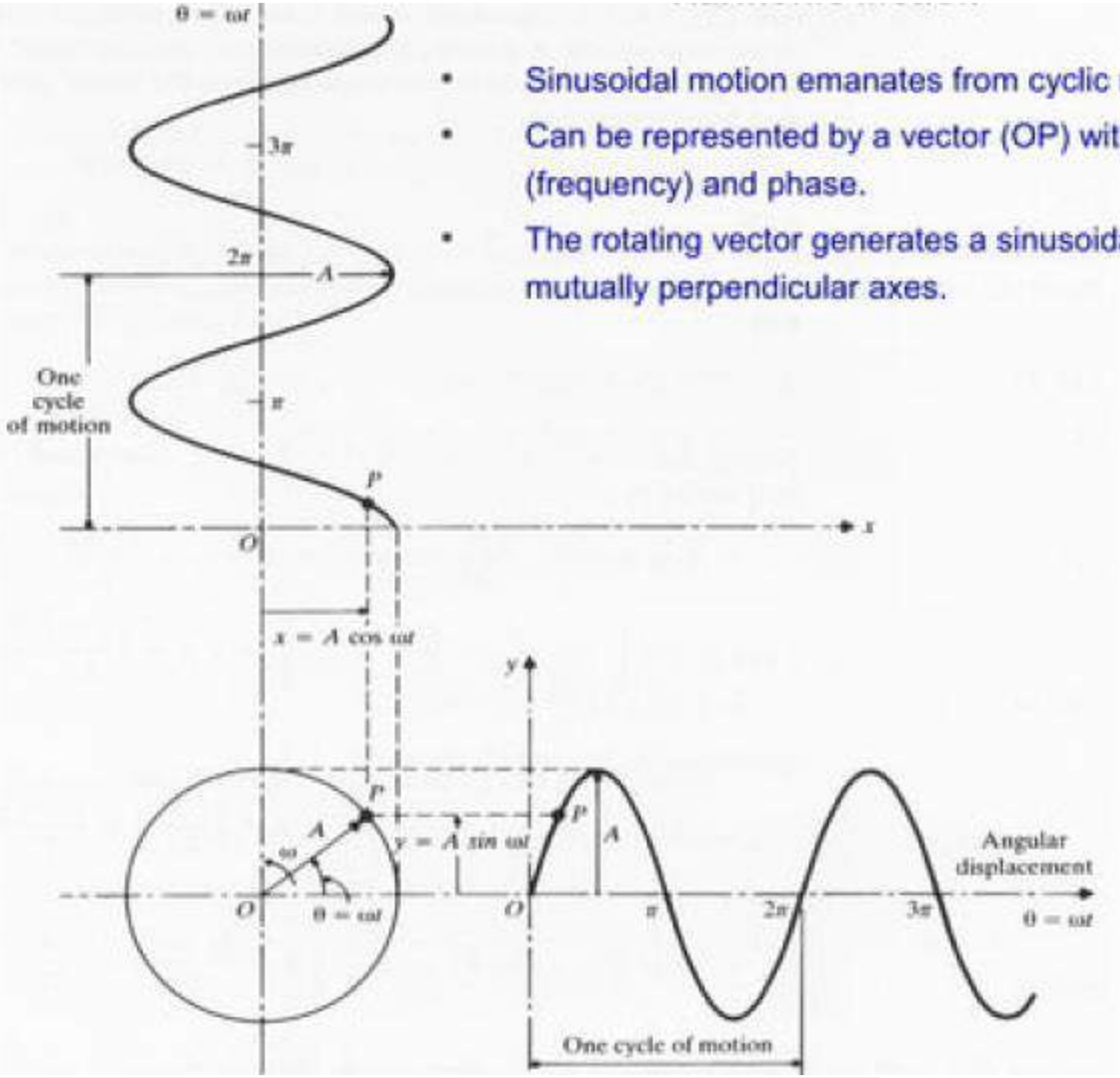
and

$$\frac{d^2x}{dt^2} = -\omega^2 A\sin(\omega t) = -\omega^2 x$$



Harmonic Motion

- Sinusoidal motion emanates from cyclic motion
- Can be represented by a vector (OP) with a magnitude, angular velocity (frequency) and phase.
- The rotating vector generates a sinusoidal and a co-sinusoidal components along mutually perpendicular axes.



Some Definitions

- **Cycle**: motion of body from equilibrium position → extreme position → equilibrium position → extreme position in other direction → equilibrium position .
- **Amplitude**: Maximum value of motion from equilibrium. (Peak – Peak = 2 x amplitude)
- **Period**: Time taken to complete one cycle

$$\tau = \frac{2\pi}{\omega}$$

ω = circular frequency

- **Frequency**: number of cycles per unit time.

$$f = \frac{1}{\tau} = \frac{\omega}{2\pi}$$

ω : radians/s   f Hertz (cycles /s)

- **Phase angle**: the difference in angle (lead or lag) by which two harmonic motions of the same frequency reach their corresponding value (maxima, minima, zero up-cross, zero down-cross)

Some Definitions

- **Natural frequency:** the frequency at which a system vibrates without external forces after an initial disturbance. The number of natural frequencies always matches the number of DoF.
- **Beats:** the effect produced by adding two harmonic motions with similar (close) frequencies.

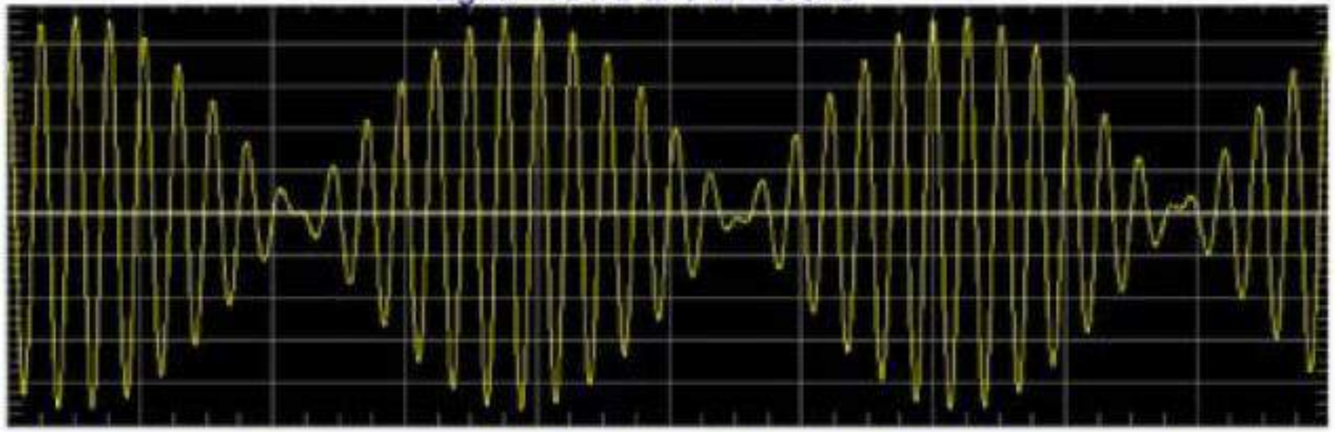
$$x_1 = A\sin(\omega t) \qquad x_2 = A\sin(\omega t + \delta\omega t)$$

$$x_t = x_1 + x_2 = A\,[\sin(\omega t) + \sin(\omega t + \delta\omega t)]$$

$$\text{Since} \qquad \sin M + \sin N = 2\sin\frac{M+N}{2}\cos\frac{M-N}{2}$$

$$x_t = 2A\sin\left(\omega t + \frac{\delta\omega t}{2}\right)\cos\left(\frac{\delta\omega t}{2}\right)$$

Eg: ω=40 Hz and δ= -0.075



- In mechanical vibratory systems, beats occur when the (harmonic) excitation (forcing) frequency is close to the natural frequency.

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Harmonic Fourier Analysis

- As for simple harmonic motion, Fourier series can be expressed with complex numbers:

$$e^{i\omega t} = \cos(\omega t) + i\sin(\omega t)$$

$$e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t)$$

$$\cos(\omega t) = \frac{e^{i\omega t} + e^{-i\omega t}}{2}$$

$$\sin(\omega t) = \frac{e^{i\omega t} - e^{-i\omega t}}{2i}$$

- The Fourier series:

$$x(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)]$$

Can be written as:

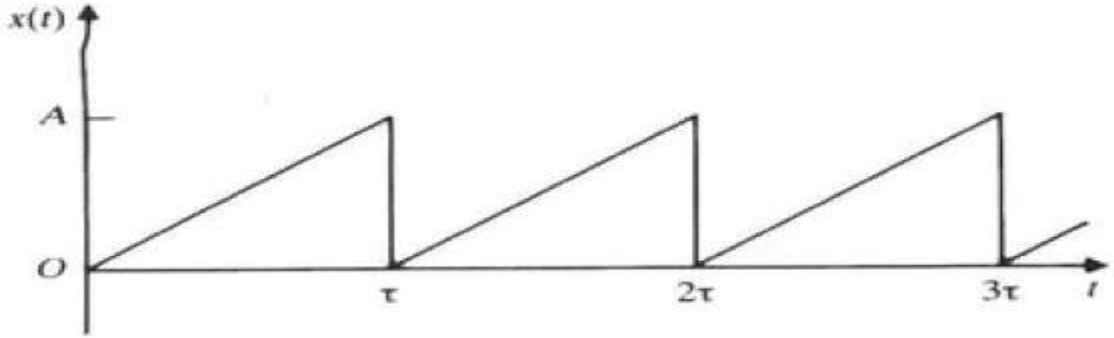
$$x(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} \left\{ a_n \left(\frac{e^{i\omega t} + e^{-i\omega t}}{2} \right) + b_n \left(\frac{e^{i\omega t} - e^{-i\omega t}}{2i} \right) \right\}$$

Harmonic Fourier Analysis

- Many vibratory systems not harmonic but often periodic
- Any periodic function can be represented by the Fourier series – infinite sum of sinusoids and co-sinusoids.

$$x(t) = \frac{a_o}{2} + a_1 \cos(\omega t) + a_2 \cos(2\omega t) + \ldots\ldots\ldots$$
$$+ b_1 \sin(\omega t) + b_2 \sin(2\omega t) + \ldots\ldots.$$
$$= \frac{a_o}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)]$$

- To obtain a_n and b_n the series is multiplied by *cos(nωt)* and *sin(nωt)* respectively and integrated over one period.
  - Example:



Harmonic Fourier Analysis

- Defining the complex Fourier coefficients

$$c_n = \frac{a_n - ib_n}{2} \quad and \quad c_{n-1} = \frac{a_n + ib_n}{2}$$

- The (complex) Fourier series is simplified to:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n \, e^{in\omega t}$$

$$x(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\omega t) + b_n \sin(n\omega t)]$$

- The Fourier series is made-up of **harmonics.**
- Their amplitudes and phases are defined as:

$$A_n = \sqrt{(a_n^2 + b_n^2)}$$

$$\phi_n = a\tan\left(\frac{b_n}{a_n}\right)$$

harmonics